the colored noise by the applied wavelet method with weights homogeneous in space and dependent on scale at one hand, and to keep in mind the difficulty in determining proper relative weights to respective data on the other hand. In addition, we should remember the fact that it is not possible to fully rely on the potential data at the large wavelet scales because they also include their own imperfection.

5. Application over Japan

We finally apply the method to real data over Japan: a high-resolution local gravity model from Kuroishi and Keller (2005) and a spherical-harmonics model of the geopotential, EIGEN-GL04S, complete to degree and order 150 (Biancale *et al.*, 2005).

The geopotential model is developed only from GRACE and LAGEOS measurements. Its cumulative error at degree 120, in terms of RMS, amounts to about 2.5 m^2/s^2, corresponding to about 25 cm in geoid height error. We calculate 5448 potential values from the model up to degree 120, regularly spaced on the ground level in the same area as that of the synthetic validation.

The local gravity model is based on a combination of land and marine gravity data and satellite altimetry derived gravity anomalies from KMS2002 (Andersen and Knudsen, 1998). We decimate it on a grid of 3 by 3 minutes and take 103,041 Faye anomalies on the ground level. The geographical distribution of Faye anomaly is shown in Fig. 10. Highest frequency undulations below 10 km of wavelength have been damped by a moving-average filter before applying the wavelet analysis.

We subtract low-degree components of EIGEN-GL04S from both data sets and apply to those data sets the method with the same parameter setting as those of the validation tests. The iteration scheme employed is one FMG-cycle followed by two V-cycles. The process is repeated by using progressively updated data a few times until convergence. Judgment of convergence in iteration is generally made in such a way that the correction increments or the ratio of them to the estimated parameters becomesmaller than the pre-assigned limits (criteria). In the case that the noises are not random or not well-behaved, however, the iterative computation becomes unstable at some stage, resulting in divergence. In such a case, we either stop the iteration computation and use the results at next-to-last step, or increase the weights of the regularization term and rework the iteration again.

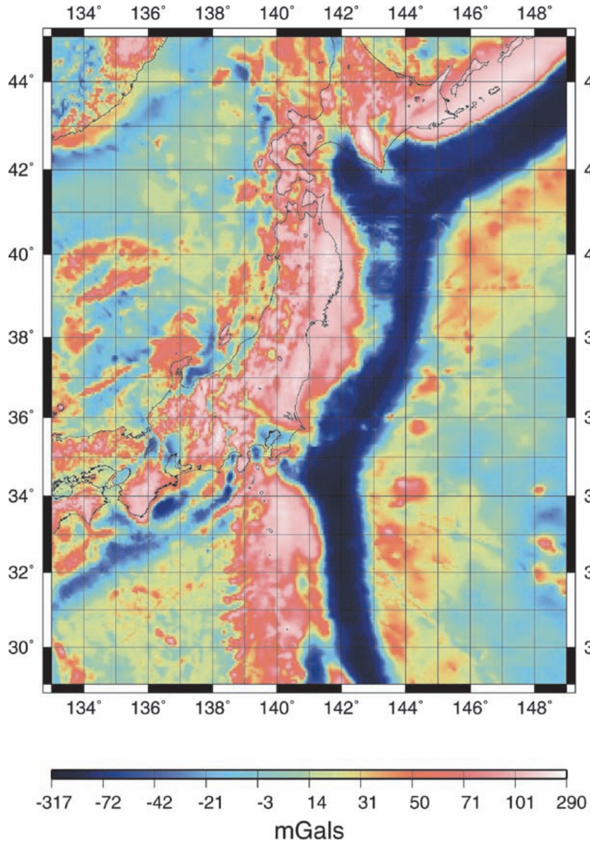

Fig. 10 Local gravity anomaly model for Japan from Kuroishi and Keller (2005).

The residuals of the respective data are shown geographically in Fig. 11. The RMSs of the residuals are 1.10 m^2/s^2 with a bias of - 0.25 m^2/s^2 for the potential data, and 1.00 mGals with a bias of 0.60 mGals at 15 km resolution for the anomaly data. These RMSs are reasonable in consideration of the data precision. Systematic residuals at large scales are remarkable particularly south of the Japanese islands and obvious along the coastal areas of the main island, Honshu as well.

These features at the ocean in the residuals are likely to reflect the systematic errors in the anomaly data controlled by the altimetric model. Kuroishi (2009) shows similar results by comparing the local gravity model with a GRACE-based geopotential model and develops a

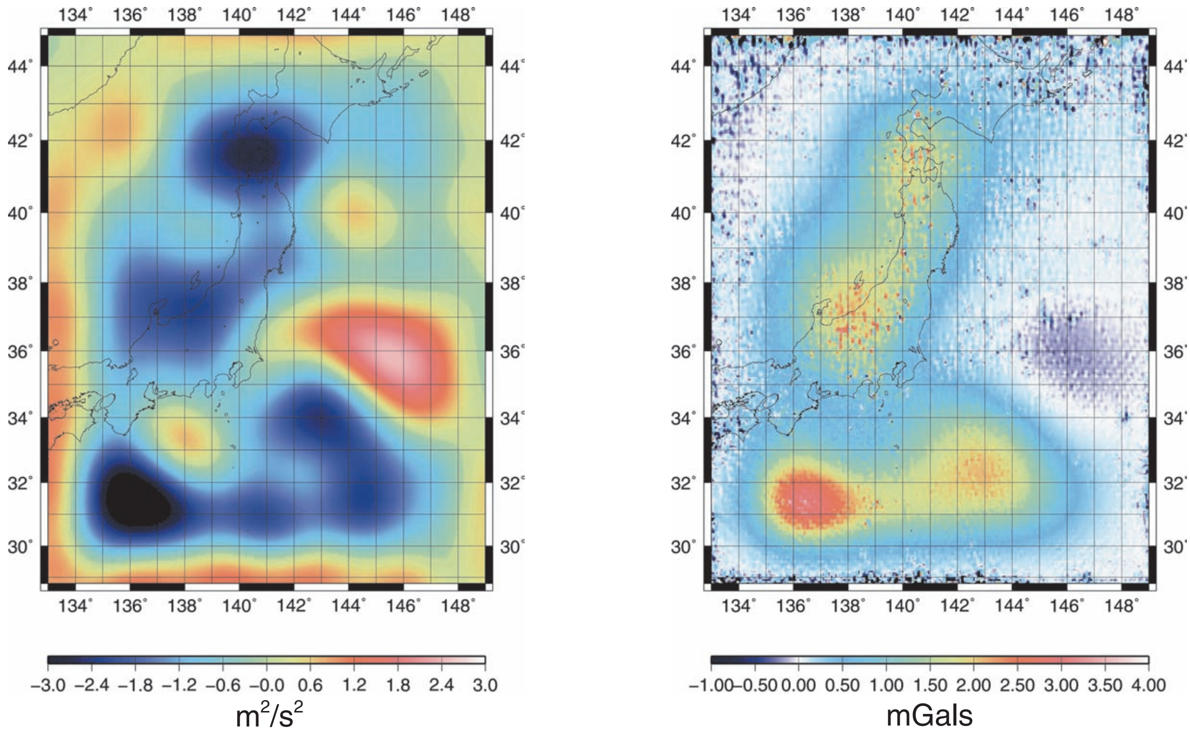

Fig. 11 Geographical distribution of residuals. *Left panel*: potential residuals to degree 120. *Right panel*: gravity anomalies at 15 km resolution

highly improved gravimetric geoid model for Japan, JGEOID2008, after removal of such errors from the local gravity model. This demonstrates how the uniformity of accuracy of the GRACE-derived static gravity model contributes much to detection of areas of degraded quality in the local gravity data.

Based on the discussion, we try to correct the anomaly data for an improved combination. First, we exercise a low-pass filter to the anomaly residuals at the resolution of EIGEN-GL04S. The left panel in Fig. 12 shows the model corrector obtained, which is subtracted from the anomaly data. Then we apply again the developed method to the corrected data sets with increased weights of the potential data with respect to the anomaly data. By inspecting the post-fit residuals, we progressively refine the model corrector and repeat the method for combination.

The final model corrector thus obtained is shown in the right panel in Fig. 12. The corrector is subtracted from the anomaly data and the corrected data are used for combination by the method developed. The residuals of the geopotential data and of the corrected anomaly data are represented geographically in Fig. 13. The RMSs of residuals are 0.40 m^2/s^2 with a bias of -0.01 m^2/s^2 for the potential data, and 0.45 mGals with a bias of 0.006 mGals at 15 km resolution for the corrected anomaly data. We find that no significant bias remains in both residuals and these RMSs are well below the estimated levels of data noise.

In the plot of the residuals of the corrected anomaly data, on the right panel in Fig. 13, features only on quite small scales are dominant. This indicates that the resolution of the combined wavelet model is a little coarser than that of the anomaly data.

In addition, we observe some edge effects, especially in the northern and southern boundaries in the case of the anomaly data. The same tendency is also visible in the case of the potential data. This shows that the inversion slightly lacks stability in these areas.

6. Conclusion

We have developed an iterative method to combine various kinds of gravity data into a wavelet model of the geopotential. This method was validated with synthetic data and then applied to real data over Japan: local high-resolution gravity anomaly data and a GRACE-derived global model, EIGEN-GL04S. We obtained a hybrid spherical harmonics/wavelet model of

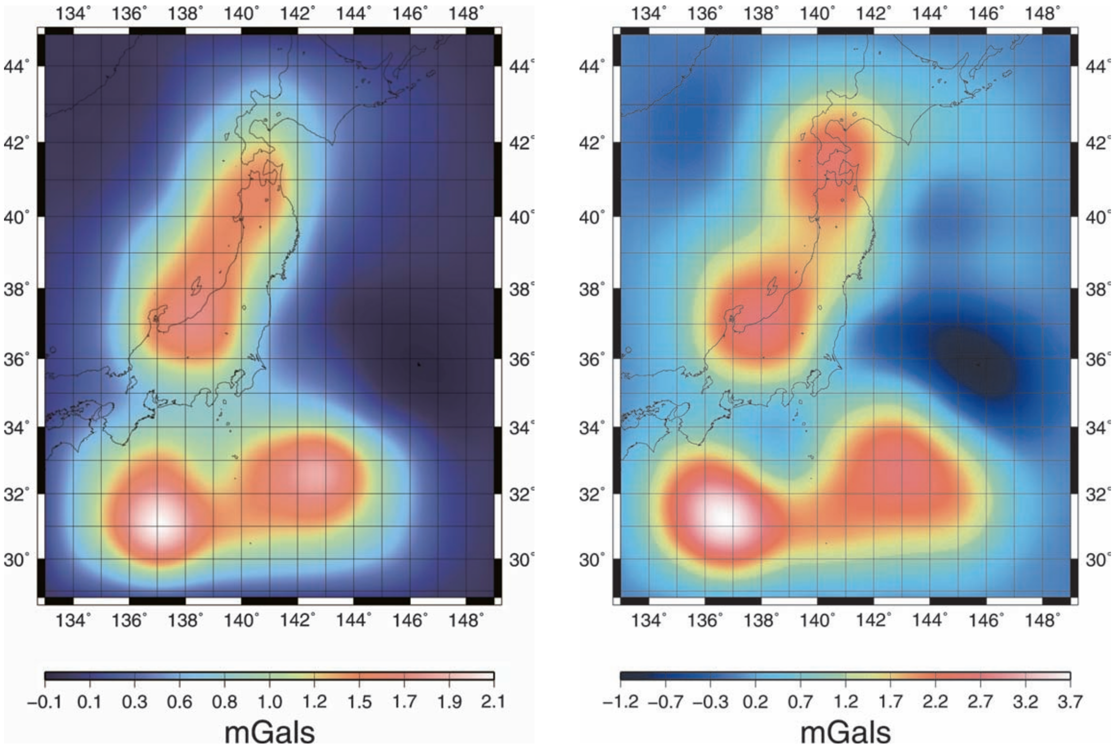

Fig. 12 Correctors to the local gravity anomaly model. *Left panel*: corrector estimated after a first series of iteration, *Right panel*: the final corrector used

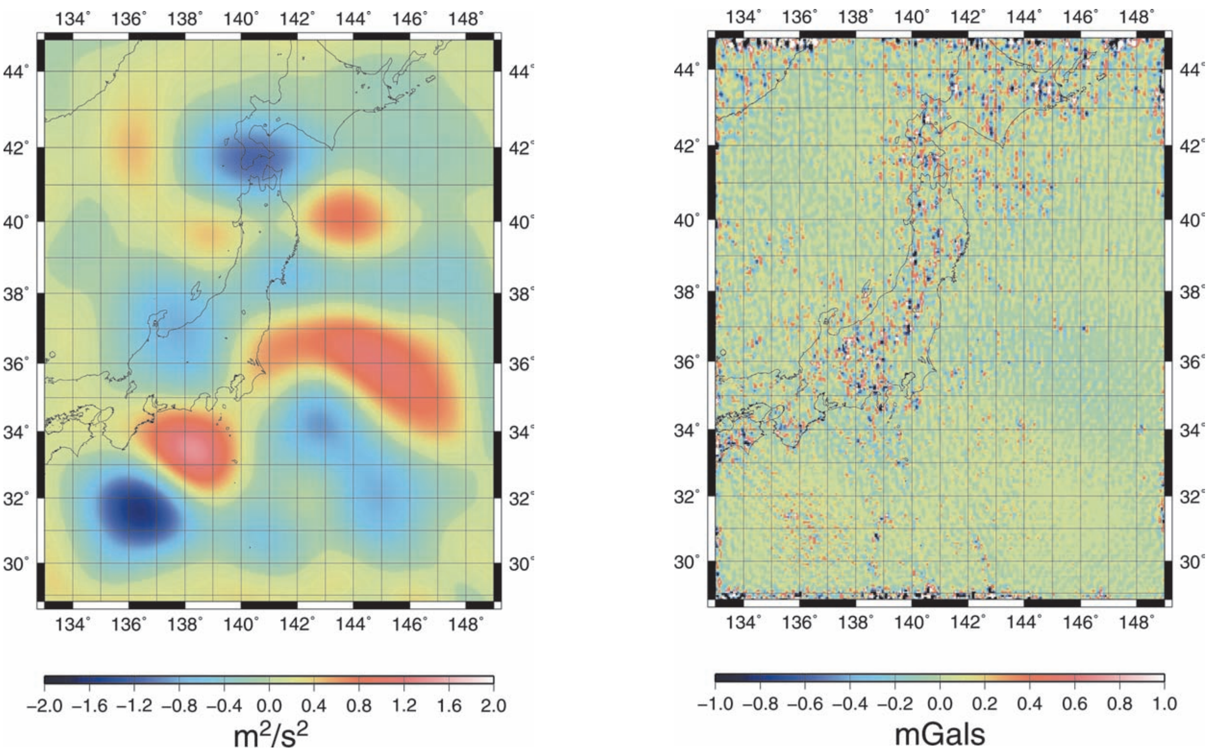

Fig. 13 Geographic distribution of residuals in the final combination. *Left panel*: potential residuals to degree 120. *Right panel*: residuals of corrected gravity anomalies at 15 km resolution

the geopotential over Japan at about 15 km resolution and the residuals of the respective data underlined biases on medium scales between the two data sets, whose suspected origin is in errors of the anomaly data. We then corrected the anomaly data by subtracting the evidenced biases and repeated the method again to the corrected data sets, resulting in an improved hybrid model. The method is applicable to directly handle satellite observation data instead of a global spherical harmonics model, which should better constrain the geopotential model at medium wavelengths. We intend to work on that in the future.

References

Andersen, O.B. and P. Knudsen (1998): Global marine gravity field from the ERS-1 and Geosat geodetic mission altimetry, Journal of Geophysical Research, 103, 8129-8137.

Biancale, R., J-M. Lemoine, G. Balmino, S. Loyer, S. Bruisma, F. Perosanz, J-C. Marty and P. Gegout (2005): Three years of decadal geoid variations from GRACE and LAGEOS data, CD-Rom, CNES/GRGS product.

Chambodut, A., I. Panet, M. Mandea, M. Diament, O. Jamet, and M. Holschneider (2005): Wavelet frames: an alternative to the spherical harmonics representation of potential fields, Geophysical Journal International, 168, 875-899.

Chan, T. and T. Mathew (1994): Domain decomposition algorithms, Acta Numerica, 61-143.

Ditmar, P., R. Klees and F. Kostenko (2003): Fast and accurate computation of spherical harmonics coefficients from satellite gravity gradiometry data, Journal of Geodesy, 76, 690-705.

Engl, H.W. (1987): On the choice of the regularization parameter for iterated Tikhonov regularization of ill-posed problems, Journal of Approximation Theory, 49, 55-63.

Freeden, W., T. Gervens and M. Schreiner (1998): Constructive Approximation on the Sphere (With Applications to Geomathematics), Oxford Science Publication, Clarendon Press, Oxford.

Frommer, A. and D. Szyld (1998): Weighted max norms, splittings and overlapping additive Schwarz iterations, Research BUGHW-SC 98/3, Bergische Universität GH Wuppertal.

Holschneider, M. (1995): Wavelets: an analysis tool, Oxford Sciences Publications, Oxford.

Holschneider, M., A. Chambodut and M. Mandea (2003): From global to regional analysis of the magnetic field on the sphere using wavelet frames, Physics of the Earth and Planetary Interiors, 135, 107-124.

Ilk, K.H., J. Kusche and S. Rudolph (2002): A contribution to data combination in ill-posed downward continuation problems, Journal of Geodynamics, 33, 75-99.

Kaula, W.M. (1966): Theory of satellite geodesy, Waltham, Blaisdell.

Klees, R., R. Koop, J. Visser and J. Van der Ijssel (2000): Efficient gravity field recovery from GOCE gravity gradient observation, Journal of Geodesy, 74, 561-571.

Kuroishi, Y. (2009): Improved geoid model determination for Japan from GFRACE and a regional gravity field model, Earth, Planets and Space, accepted for publication.

Kuroishi, Y. and W. Keller (2005): Wavelet improvement of gravity field-geoid modeling for Japan, Journal of Geophysical Research, 110, B03402, doi:10.1029/2004JB003371.

Kusche, J. (2000): Implementation of multigrid solvers for satellite gravity anomaly recovery, Journal of Geodesy, 74, 773-782.

Mallat, S. (1999): A wavelet tour of signal processing, 2nd edition, Academic Press, San Diego.

Minchev, B., A. Chambodut, M. Holschneider, I. Panet, E. Scholl, M. Mandea and G. Ramillien (2008): Local multipolar expansions for potential fields modeling, Earth, Planets and Space, submitted.

Moritz, H. (1989): Advanced physical geodesy, 2nd edition, Wichmann, Karlsruhe.

Panet, I., A. Chambodut, M. Diament, M. Holschneider and O. Jamet (2006): New insights on intraplate volcanism in French Polynesia from wavelet analysis of GRACE, CHAMP and sea-surface data, Journal of Geophysical Research, 111, B09403, doi:10.1029/2005JB004141.

Panet, I., O. Jamet, M. Diament and A. Chambodut (2004): Modelling the Earth's gravity field using wavelet frames, Proceedings of the Geoid, Gravity and Space Missions 2004 IAG Symposium, Porto.

Pavlis, N., S. A. Holmes, S. C. Kenyon and J. K. Factor (2008): An Earth gravitational model to degree 2160: EGM2008, presented at the 2008 General Assembly of the European Geosciences Union, Vienna, April 13-18.

Schwintzer, P. (1990): Sensitivity analysis in least squares gravity field modelling by means of redundancy decomposition of stochastic a-priori information, Internal Report PS/51/90/DGFI, DGFI Dept. 1, Munich.

Wesseling, P. (1991): An introduction to multigrid methods, John Wiley & Son, New York.

Xu, J. (1992): Iterative methods by space decomposition and subspace correction, SIAM Review, 34, 4, 581-613.